19 Semantics and Logic

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Tarski's seminal work on truth and logical consequence is, perhaps, the single most important contribution to modern semantics. The recursive definition of truth in terms of satisfaction and the inductive, step-by-step definition of the logical syntax on which it is based, the notion of semantic model, the definitions of logical truth and logical consequence, are at the core of contemporary semantic theories.[1] Model-theoretic semantics (abstract logic), possible-world semantics, theories of meaning such as Davidson's and others', Montague semantics and even logical form (LF), a branch of generative syntax, all incorporate Tarskian principles.[2] Tarski's theory, however, is a logical semantics, resting, as it does, on an essential division of terms into logical and extra-logical. In the mid-fifties a generalization of logical terms led to a substantial expansion of Tarskian semantics with important advances in logic and, more recently, in linguistics. Philosophically, the new generalization has raised, as well as provided tools for answering, important questions about logical semantics and its relation to linguistic semantics. In this paper I will discuss Tarskian semantics and the generalizations that followed in an attempt to answer some of the ensuing philosophical questions.

1 Tarskian Semantics Before 1957

In "The concept of truth in formalized languages" (1933) Tarski observed that the task of defining truth for a language with infinitely many sentences is complicated. We cannot refer to infinitely many sentences directly in a finite discourse, hence an indirect method is called for. The recursive method naturally suggested itself. The recursive method allows one to define predicates in a finite manner provided the domain of objects over which they range is itself finitely definable in a certain specified sense. In particular, if we construct the domain of objects inductively – i.e. generate it from a directly specified "base" (a non-empty set of objects) by means of a finite number of operations – if, furthermore, the domain is freely generated by this construction (i.e., an object cannot be constructed in more than one way), and if, finally, we limit ourselves to predicates whose satisfaction is a matter of the inductive structure of the objects involved, then we can define the extension of these predicates over the given domain recursively. Given a set S with a base B and, say, two freely generating operators, f^1 and g^2, the recursive definition of a predicate $\mathcal{P}$ (i) specifies for each atomic element whether it satisfies $\mathcal{P}$, (ii) provides a recursive rule for each generative operator: a rule that shows whether $f(a)$ satisfies $\mathcal{P}$ based on whether a satisfies $\mathcal{P}$, and a rule that shows whether $g(a,b)$ satisfies $\mathcal{P}$ based on whether a, b satisfy $\mathcal{P}$.[3] In Tarskian semantics the domain of objects is a set of formulas, and complex formulas are generated from simpler ones by means of certain logical operators. Another logical term (identity) participates in the construction of atomic formulas, although atomic formulas do not, in general, require logical terms. The definition of truth (in terms of satisfaction) specifies how the truth (satisfaction) of a complex formula is determined by the truth (satisfaction) of its constituent subformulas and the logical operators involved. The basic, non-recursive clause includes a specific rule for atomic formulas containing a logical term (identity) as a constituent, and a general rule for all other atomic formulas. Briefly, the definitions of the syntax and the semantics go as follows (I restrict myself here to 1st-order languages with no functional constants):

1.1 Syntax

Let L be a 1st-order language. We distinguish L by its non-logical constants: individuals, $c_1, \ldots, c_j$, and k_i-place predicates, $\mathcal{P}_1^{k_1}, \ldots, \mathcal{P}_m^{k_m}$, where $j, m \geq 0$ and $k > 0$. (The variables, logical symbols and punctuation marks (brackets) are the familiar ones, and they are common to all 1st-order languages.)

1.1.1 *Inductive definition of well-formed formula (wff) of* L

Base:

- If $\mathcal{P}^n$ is a non-logical predicate constant and $t_1, \ldots, t_n$ are individual terms (individual constants or variables), then $\ulcorner \mathcal{P}t_1, \ldots, t_n \urcorner$ is a wff.
- If t_1, t_2 are individual terms, then $\ulcorner t_1 = t_2 \urcorner$ is a wff.

Inductive clauses:

- If Φ is a wff, then $\ulcorner \sim\Phi \urcorner$ is a wff.
- If Φ, Ψ are wffs, then $\ulcorner \Phi \vee \Psi \urcorner$ is a wff.
- If Φ is a wff and x is an individual variable, then $\ulcorner \forall x \Phi \urcorner$ and $\ulcorner \exists x \Phi \urcorner$ are wffs.
- Only expressions obtained by one of the base or inductive clauses above are wffs.

1.2 Semantics for L

1.2.1 Recursive definition of satisfaction for L (a modern version):

Let A be the universe of discourse (a non-empty set of individuals) and d a denotation function which assigns to each individual constant c of L a member of A and to each non-logical n-place predicate constant $\mathcal{P}^n$ of L a subset of A^n (an n-place relation on A or a subset of A when $n = 1$). Let G be the collection of all functions from the set of variables of L into A. For each $g \in G$ let $\bar{g}$ be its extension to all the individual terms t of L: $\bar{g}(t) = g(t)$ if t is a variable, $\bar{g}(t) = d(t)$ if t is a constant. Then, relative to A and d, satisfaction of a wff Φ by a $g \in$ G – or, as I will present it here, the *truth value of Φ under g*, $v(\Phi[g])$, (where $v(\Phi[g]) \in \{T,F\}$) – is defined recursively as follows:

Base:

- $v(\ulcorner \mathcal{P}t_1, \ldots, t_n \urcorner[g]) = T$ iff (if and only if) $\langle \bar{g}(t_1), \ldots, \bar{g}(t_n) \rangle \in d(\mathcal{P})$.
- $v(\ulcorner t_1 = t_2 \urcorner[g]) = T$ iff $\bar{g}(t_1)$ is the same as $\bar{g}(t_2)$.

Recursive clauses:

- $v(\ulcorner \sim\Phi \urcorner[g]) = T$ iff not $v(\Phi[g]) = T$.
- $v(\ulcorner \Phi V \Psi \urcorner[g]) = T$ iff $v(\Phi[g]) = T$ or $v(\Psi[g]) = T$.
- $v(\ulcorner \forall x\Phi \urcorner / \ulcorner \exists x\Phi \urcorner[g]) = T$ iff for every/some individual a in A, $v(\Phi[g(a/x)]) = T$, where $g(a/x)$ assigns a to x and otherwise is the same as g.

1.2.2 Definition of truth:

A sentence (closed wff) σ is <u>true</u> (relative to A,d) iff its truth value under every (or some) $g \in G$ (it comes to the same thing) is T.

In order to define the notions of <u>logical truth</u> and <u>logical consequence</u>, Tarskian semantics introduces an apparatus of models. Given a 1st-order language L (as above), a *model* $\mathfrak{A}$ for L is a pair $\langle A,d \rangle$ where A is any universe (non-empty set of objects) and d a denotation function defined relative to A as above. The definitions of truth and satisfaction in a model are also the same as above, only now A is the universe of the given model and d its denotation function.

1.2.3 Definitions of logical truth and consequence:

- A sentence σ of L is a *logical consequence* of a set Γ of sentences of L iff there is no model $\mathfrak{A}$ for L in which all the sentences of Γ are true and σ is false.
- A sentence σ of L is *logically true* iff it is a logical consequence of any set of sentences of L (equivalently: iff it is true in every model $\mathfrak{A}$ for L.)

2 Unanswered Questions

Tarski's work in semantics led to a torrent of philosophical writings, but several important questions concerning the nature of logical semantics and semantics in general were rarely raised. In particular, the role, scope and nature of logical terms in Tarskian semantics as well as the relation between logical and non-logical semantics were never adequately clarified.

(A) The role of logical terms in Tarskian semantics is critical. The construction of the syntax as well as the definition of truth via satisfaction are based on fixed functions that correspond to particular constants – identity, truth-functional connectives, the existential and/or universal quantifiers. It is usually taken for granted that these constants are logical – that, furthermore, these are all the logical constants there are. With the exception of Tarski, the early logicians, and many of their successors, were content with this state of affairs. No one proposed a direct justification for his choice of quantifiers, though some produced indirect justifications. Thus, Frege and Russell believed that all of mathematics is reducible to logic with the "standard" logical terms, and Quine (perhaps the most influential of the later philosophers of logic) believed the "remarkable concurrence" of the semantic and proof-theoretic definitions of logical truth and consequence in "standard" 1st-order logic – completeness – justified drawing the boundary in the standard way. But the questions naturally arise whether mathematics cannot be reduced to logic with other logical terms, and whether no other (interesting) logical systems are complete. (Indeed, we now know that some interesting non-standard systems are.[4]) Frege referred to the standard quantifiers as expressions of generality, and in many textbooks logic is characterized as "general" and "topic neutral" and logical truths as "necessary." But in the absence of adequate criteria for generality, topic neutrality and necessity, why should we think that "not," "or," "is," "all" and their derivatives are the only carriers of general, topic neutral, necessary truths? Moreover, why should we think that the attributes of generality, topic neutrality and necessity uniquely identify logical truth at all (rather than a more inclusive – or a narrower – category of truths)?

Already in 1936 Tarski realized the issue of logical terms is crucial for logical semantics:

> Underlying our whole construction [of the semantic definition of logical consequence] is the division of all terms of the language discussed into logical and extra-logical. This division is certainly not quite arbitrary. If, for example, we were to include among the extra-logical signs the implication sign, or the universal quantifier, then our definition of the concept of consequence would lead to results which obviously contradict ordinary usage. On the other hand, no objective grounds are known to me which permit us to draw a sharp boundary between the two groups of terms. It seems to me possible to include among logical

> terms some which are usually regarded by logicians as extra-logical without running into consequences which stand in sharp contrast to ordinary usage. In the extreme case we could regard all terms of the language as logical. (Tarski 1936a: 418–9)

The extreme case Tarski referred to is one in which the boundary between logical and material consequences largely disappears. Such materially valid arguments as "Bush lost the 1992 US presidential elections; therefore, Clinton is the US president in 1994" come out logically valid. To avoid this result, a reasoned distinction between logical and extra-logical terms has to be established. Clearly, the task of establishing such a distinction is of utmost importance for logic. For many years, however, philosophers of logic took the standard division of terms into logical and extra-logical as given. This was not so much a matter of choice, but a matter of not knowing how to go about establishing a reasoned distinction.

(B) The Tarskian definition of truth in the form given above (or in any of the common forms) is inherently uninformative: Essentially, what this definition says is that $\ulcorner\Phi$ or $\Psi\urcorner$ is true iff Φ is true or Ψ is true, $\ulcorner$Some x is $\Phi x\urcorner$ is true iff some individual in the universe satisfies $\ulcorner\Phi x\urcorner$, and so on. If "or" in $\ulcorner\Phi$ or $\Psi\urcorner$ or "some" in $\ulcorner$Some x is $\Phi x\urcorner$ is unclear, ambiguous, or imprecise, the above definition of truth does not assist us in clarifying, disambiguating, or rendering it precise. For the connectives, however, we do have an informative definition available, tied up with a precise criterion of logicality. In the early days of modern logic the distinctive feature of logical connectives was determined to be truth-functionality, and, based on this feature, logical connectives were identified with certain mathematical functions, namely Boolean truth functions (functions from sequences of truth values to truth values): Negation was identified with the 1-place function $f_{\sim}$, where $f_{\sim}(T) = F$ and $f_{\sim}(F) = T$, disjunction was identified with the 2-place function, $f_{\vee}$, where $f_{\vee}(T,T) = f_{\vee}(T,F) = f_{\vee}(F,T) = T$ and $f_{\vee}(F,F) = F$, etc. The semantic identification of logical connectives with Boolean functions led to the following criterion for logical connectives:

(LC) A term C is a logical connective iff there is a natural number n and a Boolean function $f_c:\{T,F\}^n \rightarrow \{T,F\}$ such that for any n-tuple of well-formed sentences $\sigma_1, \ldots, \sigma_n$, $\ulcorner C(\sigma_1, \ldots, \sigma_n)\urcorner$ is a well-formed sentence and its truth value is determined by $f_c[v(\sigma_1), \ldots, v(\sigma_n)]$, where for $1 \leq i \leq n$, $v(\sigma_i)$ is the truth value of σ_i.

This criterion gives a precise and informative answer to the questions: "What is a logical connective?", "What are all the logical connectives?". It decides the adequacy of a given selection of logical connectives (~ and V constitute a "complete" selection but & and V do not: we can define all truth-functional connectives in terms of ~ and V, but not in terms of & and V). And it enables us to give a more informative account of the truth conditions of sentential compounds:

- $v(\ulcorner \sim \Phi \urcorner [g]) = T$ iff $f_{\sim}(v(\Phi[g])) = T$,
- $v(\ulcorner \Phi V \Psi \urcorner [g]) = T$ iff $f_{v}(v(\Phi[g]),v(\Psi[g])) = T$.

Whereas in the earlier version the definiens simply simulated the definiendum, here the definiens includes a precise and informative rendition of the latter. (Thus, if the colloquial "or" is ambiguous between the exclusive and inclusive reading, the Boolean function associated with "or" resolves the ambiguity.)

Unlike the logical connectives, the logical predicates and quantifiers are usually defined by enumeration and the meta-theoretical account does little more than translate them into colloquial language. It is true that their frequent use in mathematics has made these terms precise, but no systematic identification of the logical predicates and quantifiers with mathematical functions based on a general criterion of logicality is given.

(C) Tarski's recursive definition of truth is limited to sentences generated by means of *logical* operators. In this definition each logical term receives special treatment, but all non-logical terms of a given grammatical category are treated "en masse." The question naturally arises whether Tarskian semantics is inherently logical. Clearly, in logic, we are interested in studying the contribution of logical structure to the truth value of sentences, but logical structure is not the only factor in the truth or falsity of sentences. To what extent is Tarski's method limited to logical semantics? Should we think of natural language semantics as a straightforward extension of Tarski's theory?

These questions, thus, are awaiting an answer: (A) Is there a philosophical basis for the distinction between logical and non-logical terms? (B) Can we develop a precise mathematical criterion for logical constants and, based on it, an informative definition of truth? (C) What is the relation between logical and general semantics? Before 1957 it was hard to answer the first two questions since no systematic study of logical terms (other than connectives) existed. But the generalization of the standard quantifiers by Mostowski (1957) changed this situation: it created a framework within which to develop, compare and investigate alternative answers to questions about logical terms.

3 Mostowski's Generalization of the Logical Quantifiers and Further Developments

In his 1957 paper, "On a generalization of quantifiers," Mostowski proposed a semantic criterion for logical quantifiers leading to a new notion of logical term, considerably broader than the standard one. Before I describe Mostowski's criterion I would like to introduce a syntactic–semantic classification of primitive terms that is independent of their logical status. Relative to this classification we will be able to view Mostowski's generalization as a first step in answering the question: "What terms of what categories are logical, and why?"

Adopting a Fregean conception of quantifiers as properties (relations) of properties (relations) of individuals, I will present a syntactic classification of terms into *orders* and *types* based on semantic considerations. The order of a term depends on whether its extension or denotation (in what may be called its "intended model") is an individual, a set or n-tuple of individuals, a set of sets of individuals, etc. (I assume the notion of an empty set of individuals is distinguished from that of an empty relation of individuals, an empty set of sets of individuals, etc.) I am leaving functional terms and sentential connectives out, the former for the sake of simplicity, the latter because the problem of providing a precise criterion of logicality for the connectives has its own, independent, solution. (See above)

Order:

- A primitive individual term (a term denoting an individual), t. *order* 0.
- A primitive n-place predicative term (a term whose extension is a set or relation), P (where $n > 0$): *order* $m + 1$, where m is the order of the highest argument of P.

On this classification, a 1st-order system consists of primitive terms (or schematic representations of primitive terms) of orders 0, 1 and 2, where all variables are of order 0 and all primitive terms of order 2 are logical. I.e. a 1st-order system is a system whose non-logical constants and variables are of order ≤ 1. Primitive terms of order 0 are called individual constants, of order 1 – predicates, and of order 2 – quantifiers. Restricting ourselves to constants of orders 0, 1 and 2, we can determine their place in the hierarchy uniquely by means of a simple classification into "types."

Type:

The type of a primitive term provides information about its arguments.

- A primitive individual term, t. *no type* (no arguments).
- A primitive n-place predicative term, P: *type* $\langle t_1, \ldots, t_n \rangle$, where for $1 \leq i \leq n$, $t_i =$ the number of arguments of the i-th argument of P.

Predicates are of type $\langle t_1, \ldots, t_n \rangle$, where for all $1 \leq i \leq n$, $t_i = 0$; quantifiers are of type $\langle t_1, \ldots, t_n \rangle$, where for each $1 \leq i \leq n$, $t_i \geq 0$, and for at least one $1 \leq i \leq n$, $t_i > 0$. Identity is of type $\langle 0,0 \rangle$; the existential and universal quantifiers are of type $\langle 1 \rangle$. Natural language constants are naturally classified as follows: "John" – no type; "is tall" – type $\langle 0 \rangle$; "loves" – type $\langle 0,0 \rangle$. Mathematical constants receive the following classification: "one" – no type; "there is exactly one" – type ⟨1⟩; "there are finitely many" – type ⟨1⟩; "most . . . are ---" – type ⟨1,1⟩; "is a well ordering" – type ⟨2⟩; the membership predicate of 1st-order set theories (e.g. ZF) – type ⟨0,0⟩; the 2nd-order membership predicate – type ⟨0,1⟩, and so on.

Mostowski's generalization can be seen as an answer to the question: "What primitive terms of type ⟨1⟩ (i.e. quantifiers of type ⟨1⟩) are logical?" His answer is given by the following semantic criterion:

> (M) "[Logical] quantifiers should not allow us to distinguish between different elements of [the underlying universe]." (Mostowski 1957: 13)

Two questions naturally arise: (1) What is the precise content of (M)? In particular: What does "not distinguishing between different elements" come to? (2) What is the intuitive justification for viewing logical quantifiers as in (M)? Mostowski did not give an answer to (2), but he did give a precise answer to (1). I will begin with Mostowski's syntactic notion of quantifier.

Syntactically, any predicate of type ⟨1⟩ can be construed as a 1-place operator binding a formula by means of a variable, i.e. as a quantifier. A quantifier (of type ⟨1⟩) satisfying (M) is a logical quantifier. To incorporate 1-place quantifiers in a 1st-order syntax we replace the entry for quantification in the standard definition by:

- If Φ is a wff, x is a variable and Q is a logical quantifier, then $\ulcorner Qx\Phi \urcorner$ is a wff.

Disengaging ourselves from the notion of "intended model," we can view quantifiers, semantically, as functions that assign to each universe A an A-quantifier, where an A-quantifier is a set of subsets of A or a function from subsets of A to $\{T,F\}$. Thus, the universal quantifier is a function $Q^{\forall}$, or shortly, $\forall$, such that for any set of objects, A, $\forall(A) = \forall_A{:}P(A) \rightarrow \{T,F\}$, where $P(A)$ is the power set (set of all subsets) of A. $\forall_A$ is defined by: given a subset B of A, $\forall_A(B) = T$ iff $B = A$. $\exists_A$ is defined by: $\exists_A(B) = T$ iff $B \neq \phi$. Mostowski interpreted the semantic condition (M) as saying:

> (M*) An A-quantifier is logical iff it is invariant under permutations of A (or, more precisely, permutations of $P(A)$ induced by permutations of A).

I.e. Q_A is logical iff for any permutation p of A and any subset B of A, $Q_A(B) = Q_A(p'(B))$, where p' is the permutation of $P(A)$ induced by p. (M*) can also be formulated in terms of (weak) automorphisms (isomorphisms of A-structures):

> (M**) An A-quantifier is logical iff it is invariant under automorphisms of set A-structures.

I.e. Q_A is logical iff given a non-empty set A and $B, B' \subseteq A$: if $\langle A,\langle B\rangle\rangle \cong \langle A,\langle B'\rangle\rangle$ ($\langle A,\langle B\rangle\rangle$ and $\langle A,\langle B'\rangle\rangle$ are isomorphic), then $Q_A(B) = Q_A(B')$. It is easy to see that the standard quantifiers satisfy Mostowski's condition.

Now, as in sentential logic, Mostowski was able to represent the truth (satisfaction) conditions of logical A-quantifiers by correlating them with certain mathematical functions. Given a logical A-quantifier, Mostowski observed that its truth conditions have to do with the *cardinalities* of subsets of A and their complements (in A) and nothing else. Accordingly, $\exists_A$ and $\forall_A$ can be defined in the following way: Given a set $B \subseteq A$, $\exists_A(B) = T$ iff $|B| > 0$, $\forall_A(B) = T$ iff $|A - B| = 0$ ($|\ldots|$ is the cardinality of ...). We can thus identify $\exists_A$ and $\forall_A$ with *cardinality functions*, $\exists_{|A|}$ and $\forall_{|A|}$, from pairs of cardinals (β,γ) such that $\beta + \gamma = |A|$ to T and F, defined by: $\exists_{|A|}(\beta,\gamma) = T$ iff $\beta > 0$, $\forall_{|A|}(\beta,\gamma) = T$ iff $\gamma = 0$. More generally, any function $q_\alpha:(\beta,\gamma)_\alpha \rightarrow \{T,F\}$, where α is a cardinal number larger than 0 and $(\beta,\gamma)_\alpha$ is the set of all pairs (β,γ) such that $\beta + \gamma = \alpha$, is an α-quantifier. Mostowski proved that there is a 1–1 correlation between logical A-quantifiers, i.e. A-quantifiers satisfying (M*), and α-quantifiers.

How shall we extend (M*) to logical quantifiers of type $\langle 1\rangle$ in general, i.e. unrestricted to A? Mostowski did not go beyond A-quantifiers, but we can easily extend his criterion based on the following considerations:

(a) It is natural to view a quantifier Q as logical iff for any universe A, Q_A is a logical A-quantifier.
(b) It is natural to require that a logical quantifier Q be correlated with the same α-quantifier in any two universes of the same cardinality.

(a) and (b) lead to the following extension of (M**):

(M***) A quantifier Q (predicate of type $\langle 1\rangle$) is logical iff it is invariant under isomorphisms of set structures.

I.e. Q is logical iff for every $A \neq \phi$, $A' \neq \phi$, $B \subseteq A$ and $B' \subseteq A'$: if $\langle A,\langle B\rangle\rangle \cong \langle A',\langle B'\rangle\rangle$, then $Q_A(B) = Q_{A'}(B')$. Quantifiers satisfying (M***) are often called "Mostowskian" or "cardinality quantifiers." Among the many quantifiers falling under this category is M, standing for some mathematically precise rendition of "most." If "most" is taken as "more than half," then it is defined by $M_\alpha(\beta,\gamma) = T$ iff $\beta > \gamma$. Other Mostowskian quantifiers are $!\delta$, standing for "exactly δ" and defined by $!\delta_\alpha(\beta,\gamma) = T$ iff $\beta = \delta$; E, standing for "an even number of" and defined by $E_\alpha(\beta,\gamma) = T$ iff β is an even positive integer; $!1/2$, standing for "half" and defined by $!1/2_\alpha(\beta,\gamma) = T$ iff $\beta = \gamma$, and $\mathcal{F}$, standing for "finitely many" and defined by $F_\alpha(\beta,\gamma) = T$ iff $\beta < \aleph_0$. These quantifiers appear in such sentences as "Most things are different from what you think they are," "There are exactly two pennies in my pocket," "There is an even number of letters in the English alphabet," "Half the things are yours," "There are finitely many rows in a truth table," symbolized by $\ulcorner(\mathrm{M}x)\mathcal{D}x\urcorner$, $\ulcorner(!2x)\mathcal{P}x\urcorner$, $\ulcorner(\mathrm{E}x)\mathcal{L}x\urcorner$,

$\ulcorner(!1/2x)\mathcal{Y}x\urcorner$ and $\ulcorner(\forall x)[\mathcal{T}x \supset (\mathrm{F}y)\mathcal{R}yx]\urcorner$, respectively, with the obvious reading of $\mathcal{D}$, $\mathcal{P}$, $\mathcal{L}$, $\mathcal{Y}$, $\mathcal{T}$ and $\mathcal{R}$. We can replace the entry for quantifiers in Tarski's semantic definition of truth under g as follows: Given a universe A of cardinality $\alpha > 0$,

- $v(\ulcorner \mathrm{Q}x\Phi\urcorner[g]) = T$ iff $Q_\alpha(\beta,\gamma) = T$, where $\beta = |\{a \in A: v(\Phi[g(a/x)]) = T\}|$ and $\gamma = |\{a \in A: v(\Phi[g(a/x)]) = F\}|$.

Intuitively, what this definition says is that $\ulcorner \mathrm{Q}x\Phi x\urcorner$ is true in a model $\mathfrak{A}$ with a universe A of cardinality α iff the number of objects in A satisfying $\ulcorner\Phi x\urcorner$ and the number of objects in A not satisfying $\ulcorner\Phi x\urcorner$ is as Q says. Mostowski's generalization was further extended in 1966 by Lindström and Tarski (independently of one another). The extended criterion applies to all terms of orders 0–2, regardless of type. Following common practice, I will name the criterion after Lindström:

(L) A term is logical iff it is invariant under isomorphic structures,[5]

where a structure is an $n+1$-tuple, $\langle A,\langle D_1, \ldots, D_n\rangle\rangle$, $A \neq \phi$ and D_i, $1 \leq i \leq n$, is a member of A, a subset of A or a relation of A. The idea is, roughly, that if a term is logical, it does not distinguish between structurally identical arguments. I.e. if $\langle A,\langle D_1, \ldots, D_n\rangle\rangle \cong \langle A',\langle D'_1, \ldots, D'_n\rangle\rangle$, then a logical term assigns the same truth value to $\langle D_1, \ldots, D_n\rangle$ in the universe A as to $\langle D'_1, \ldots, D'_n\rangle$ in the universe A'. Since individual terms have no arguments, they cannot be said to give the same truth value to structurally identical arguments, and we stipulate that they do not satisfy the criterion (L). Among the predicates falling under Lindström's criterion are, in addition to Mostowskian quantifiers, the 1st-order identity relation and the 2nd-order binary predicate "Most" (type $\langle 1,1\rangle$), appearing in such sentences as "Most students passed the test" and symbolized $\ulcorner(\mathrm{M}^{1\text{-}1}x)(Sx,\mathcal{P}x)\urcorner$. Intuitively, $\ulcorner(\mathrm{M}^{1\text{-}1}x)(Sx,\mathcal{P}x)\urcorner$ is true iff most of the S's in the universe of discourse are P's, i.e. iff the pair $\langle S,P\rangle$, where S, P are the extensions of $\ulcorner Sx\urcorner$, $\ulcorner\mathcal{P}x\urcorner$, respectively, satisfies "Most".[6] "Most" is logical according to (L) since for any universes A and A', and $S,P \subseteq A$, $S', P' \subseteq A'$: if $\langle A,\langle S,P\rangle\rangle \cong \langle A',\langle S',P'\rangle\rangle$, then $\langle S,P\rangle$ satisfies "Most" iff $\langle S',P'\rangle$ satisfies it. Other terms satisfying (L) include the 2nd-order ternary predicate, "More . . . than --- are***" (type$\langle 1,1,1\rangle$), as in "More girls than boys passed the test," symbolized by $\ulcorner(\mathrm{M}^{1\text{-}1\text{-}1}x)(\mathcal{G}x,\mathcal{B}x,\mathcal{P}x),\urcorner$ the 2nd-order membership predicate, where "t is a member of $\mathcal{B}$" is symbolized $\ulcorner(\mathrm{MEM}^{0\text{-}1}x)(t,\mathcal{B}x)\urcorner$, the 2nd-order relational predicate "Well-ordering", where "$\mathcal{R}$ is a well ordering" is symbolized: $\ulcorner(\mathrm{WO}^2xy)\mathcal{R}xy\urcorner$, etc.

Among the predicates which do not satisfy (L) are all the predicates of type 1 which fail to satisfy (M***), the predicate "is tall", the 1st-order membership predicate, the 2nd-order predicate "is a relation between humans" (type$\langle 2\rangle$), and many others. Comparing the two membership predicates, $\in^{0\text{-}0}$ and $\in^{0\text{-}1}$, we see the difference between them intuitively as follows: $\ulcorner t_1 \in {}^{0\text{-}0} t_2 \urcorner$ is true in a universe A iff the two individuals assigned to t_1 and t_1 in A, a_1 and a_2, are such that $a_2 = \{a_1, \ldots\}$. But as members of A (as individuals) a_1 and a_2 are atomic objects. Therefore, the structures $\langle A, \langle a_1, a_2 \rangle\rangle$ and $\langle A, \langle a_2, a_1 \rangle\rangle$ are isomorphic and no term satisfying (L) distinguishes between them. I.e. for any terms of type $\langle 0,0 \rangle$, $C^{0\text{-}0}$, satisfying (L): $C_A^{0\text{-}0}(a_1, a_2) = T$ iff $C_A^{0\text{-}0}(a_2, a_1) = T$. Since $\in^{0\text{-}0}$ does distinguish between the structures $\langle A, \langle a_1, a_2 \rangle\rangle$ and $\langle A, \langle a_2, a_1 \rangle\rangle$ (if $a_1 \in_A^{0\text{-}0} a_2$ then $a_2 \notin_A^{0\text{-}0} a_1$), $\in^{0\text{-}0}$ does not satisfy (L). The situation is different with respect to $\in^{0\text{-}1}$: $\in_A^{0\text{-}1}$ holds between a member and a subset of A, not between two members of A. Therefore, the problem indicated above does not arise: $\langle A, \langle a, B \rangle\rangle$ is never isomorphic to $\langle A, \langle B, a \rangle\rangle$. It is easy to see that $\in^{0\text{-}1}$ satisfies (L): Given any non-empty sets A and A', if $\langle A, \langle a, B \rangle\rangle \cong \langle A', \langle a', B' \rangle\rangle$ then $a \in B$ iff $a' \in B'$, i.e. $\in_A^{0\text{-}1}(a, B) = T$ iff $\in_{A'}^{0\text{-}1}(a', B') = T$.

Lindström's criterion satisfies Mostowski's initial condition, (M): A term invariant under isomorphic structures takes into account only the mathematical structure of its arguments in a given universe. Since individuals are, semantically, atomic elements, they are all structurally identical, and their difference is not detected by any logical (structural) term. Taking Lindström's criterion as my starting point I will now turn to the three questions posed in Section 2. I will begin with the second, meta-logical, question: Is there an informative, constructive definition of logical terms, which exhibits their truth (satisfaction) conditions in accordance with (L) and shows how to build up extensions that satisfy them? In Section 6 I will propose an answer to the question concerning the philosophical foundation of the distinction between logical and non-logical terms, and in Section 7 I will briefly comment on the relation between logical and linguistic semantics.

4 A Constructive Definition of Logical Terms

My answer to the second, meta-logical question is positive. I will present two "constructive" definitions of logical terms: (a) a definition of n-place cardinality quantifiers, based on Lindström (1966), and (b) a definition of logical terms in general, based on Sher (1991).

4.1 *A constructive definition of n-place cardinality quantifiers*

In his 1966 paper Lindström extended Mostowski's "constructive" definition to *cardinality quantifiers* in general. Lindström proved that if a logical term of type $\langle 1, 1, \ldots, 1 \rangle$ satisfies (L), then all it takes into account is the *cardinalities* of 2^n sets in a given universe. More particularly, if a quantifier $Q^{1\text{-}1}$ satisfies (L), then the truth of "Q^{1-1} $\mathcal{B}$'s are C's"[7] is fully determined by the cardinalities of $B-C$, $C-B$, $B \cap C$ and $A-(B \cup C)$, where B and C are the extensions of $\mathcal{B}$ and C', respectively, and A is the universe of discourse. (Mathematically, the truth-conditions of Q^{1-1} are based on the size of the atoms of the Boolean algebra generated by B and C in A.) We can thus identify Q^{1-1}, semantically, with a function, Q^{1-1}, from cardinal numbers α (sizes of universes) to α-quantifiers, Q^{1-1}_α, where Q^{1-1}_α is a function from quadruples of cardinals, $(\beta, \gamma, \delta, \zeta)$, whose sum is α into $\{T,F\}$.[8] (If α represents the size of A, then $\beta,\gamma,\delta,\zeta$ represent the cardinalities of $B-C$, $C-B$, $B \cap C$ and $A-(B \cup C)$, respectively, where B and C are any subsets of A.) For example, we identify $Most^{1-1}$ with M^{1-1}, defined by: for any cardinal α, $M^{1-1}_\alpha(\beta,\gamma,\delta,\zeta) = T$ iff $\delta > \beta$. The totality of 4-place cardinality functions as above determines the totality of binary "cardinality" quantifiers, and each cardinality function embeds a set of instructions for constructing an n-tuple of sets that satisfy the corresponding logical term. (To construct a pair of sets such that $\langle B,C\rangle$ satisfying M^{1-1} in A we partition A to four subsets, A_1, A_2, A_3 and A_4 such $|A_3| > |A_1|$, and we let $B = A_1 \cup A_3$ and $C = A_2 \cup A_3$.)

We incorporate 2-place cardinality quantifiers in a 1st-order Tarskian system by adding a new entry to the syntax and the semantics:

- If Φ, Ψ are wffs, x is a variable and Q^{1-1} is a 2-place quantifier, then $\ulcorner Q^{1-1}x(\Phi,\Psi)\urcorner$ is a wff.
- $v(\ulcorner Q^{1-1}x(\Phi,\Psi)\urcorner[g]) = T$ iff $Q_\alpha(\beta,\gamma,\delta,\zeta) = T$, where $\beta = |\{a \in A\colon v(\Phi[g(a/x)]) = T$ and $v(\Psi[g(a/x)]) = F\}|$, $\gamma = |\{a \in A\colon v(\Phi[g](a/x)]) = F$ and $v(\Psi[g(a/x)]) = T\}|$, $\delta = |\{a \in A\colon v(\Phi[g(a/x)]) = v(\Psi[g(a/x)]) = T\}|$, and $\zeta = |\{a \in A\colon v(\Phi[g(a/x)]) = v(\Psi[g(a/x)]) = F\}|$.

4.2 A constructive definition of logical terms in general

Not all logical terms satisfying (L) are cardinality quantifiers, therefore a general definition of logical terms requires a different method from the one employed above. Presently, I will give an outline of such a general definition. For the purposes of demonstration I will limit myself to terms of types $\langle 0,0\rangle$, $\langle 1\rangle$ and $\langle 2\rangle$. The account will proceed from the bottom up: starting with a universe A, I will describe a method for determining (or constructing) the extensions of all logical terms in A. This method will produce a set-theoretical representation of logical terms in general. I will proceed in four steps.

Step 1. Take any model $\mathfrak{A}$ with a universe A of cardinality α and consider three sets: $S1$ – the set of all pairs of individuals in A ($S1 = A \times A$), $S2$ – the set of all subsets of A ($S2 = P(A)$), and $S3$ – the set of all sets of pairs of individuals in A ($S3 = P(A \times A)$). Subsets of $S1$ constitute extensions of terms of type $\langle 0,0\rangle$ (e.g. =) in A, subsets of $S2$ constitute extensions of terms of type $\langle 1\rangle$ (e.g. $\forall$ and $Most^1$) in A, and subsets of $S3$ are extensions of terms of type $\langle 2\rangle$ (Is-Symmetric[2]) in A. Our first task is to construct the extensions of logical terms of these types in $\mathfrak{A}$.

Step 2. We know that logical terms do not distinguish between objects (pairs of individuals, sets, relations) which are structurally identical. So we can think of logical terms in the following way: if a logical term holds of an object with structure $\mathcal{S}$ in $\mathfrak{A}$, then it holds of all objects with structure $\mathcal{S}$ in $\mathfrak{A}$. Thus, given any subset of $S1$, $S2$ or $S3$, we extend it to an extension of a logical term (of type $\langle 0,0\rangle$, $\langle 1\rangle$ or $\langle 2\rangle$) by closing it under automorphisms. For example, if $A = \{a,b,c,d\}$ and $S \subseteq S2 = \{\{a\},\{b,c\}\}$, then, since $\langle A,\langle\{a\}\rangle\rangle \cong \langle A,\langle\{b\}\rangle\rangle \cong \langle A,\langle\{c\}\rangle\rangle \cong \langle A,\langle\{d\}\rangle\rangle$ and $\langle A,\langle\{b,c\}\rangle\rangle \cong \langle A,\langle\{a,b\}\rangle\rangle \cong \langle A,\langle\{a,c\}\rangle\rangle \cong \langle A,\langle\{a,d\}\rangle\rangle \cong \langle A,\langle\{b,d\}\rangle\rangle \cong \langle A,\langle\{c,d\}\rangle\rangle$, the closure of S, $S*$, is $\{\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\}\}$. By construction, $S*$ satisfies the invariance condition (L) for A-structures (structures with A as their universe), hence $S*$ is the extension of some logical term over all models with universe A, namely the logical quantifier "either-one-or-two^1_A," or, shortly "$1\text{–}2^1_A$," whose syntactic correlate appears in formulas of the form $\ulcorner(1\text{–}2^1x)\Phi x\urcorner$. In this way the closure of each subset of $S1$, $S2$ and $S3$ constitutes the extension of some logical term restricted to A.

Step 3. Logical terms, however, satisfy the invariance condition (L) not only relative to one universe, but also across universes. Therefore, logical terms over $\mathfrak{A}$ should not be constructed from elements of A, but from "neutral" elements, elements that can be used to identify extensions in any universe of cardinality $\alpha = |A|$. One way to satisfy this requirement is to construct logical terms using indices of members of A as our building blocks. We can take α itself, i.e. the set of ordinals smaller than α, as our index set.[9] We start by assigning indices to elements of A by some (any) index function i from A *onto* α. We then replace the extensions of the logical terms over $\mathfrak{A}$ by their indices. Thus, in the example above, we assign to a,b,c,d the indices 0,1,2,3 and we replace $S*$ by $S^* = \{\{0\},\{1\},\{2\},\{3\},\{0,1\},\{0,2\},\{0,3\},\{1,2\},\{1,3\},\{2,3\}\}$. The result is an extension of an α-logical-term, in this case, a logical quantifier, $1\text{–}2_\alpha$. We say that a $B \in S2$ satisfies $1\text{–}2_\alpha$ (in $\mathfrak{A}$) iff for some indexing of A by α, the index image of B is in the extension of $1\text{–}2_\alpha$ (i.e. is a member of S^*). In a similar way we construct α-logical-terms of types $\langle 0,0\rangle$ and $\langle 2\rangle$.

Step 4. Finally, we "construct" unrestricted logical terms by grouping α-logical-terms together into classes, where each class contains exactly one α-logical-term for each cardinal α. Each class is a logical term (an unrestricted logical term), and the construction of each logical term embeds a structural description of the objects satisfying it in each model. Put otherwise, the representation of each logical term includes "instructions" for constructing its extension in a given model as well as determining, with respect to any given element (in our example, a pair of individuals, a subset of the universe, or a binary relation on the universe) whether it satisfies the given logical term in that model. Technically, I will construe both α-logical-terms and (unrestricted) logical terms as functions. A logical term assigns to each cardinal α an α-logical-term, and an α-logical-term assigns to each element of α of the right type (a pair of ordinals, a set of ordinals, a set of pairs of ordinals, etc.) a truth value, T or F. Thus, if $\alpha = 4$, $1\text{–}2^1_\alpha(\{1\}) = 1\text{–}2^1_\alpha(\{0,3\}) = T$ and $1\text{–}2^1_\alpha(\phi) = 1\text{–}2^1_\alpha(\{0,1,2\}) = F$.[10]

This completes my construction. The construction is, of course, an idealized one, using resources (i.e. proper classes) that go beyond standard set theories. But it gives us a definition analogous to the Boolean definition of the logical connectives: i.e., a definition that embeds rules for determining what satisfies a given logical term. In sentential logic logical terms do not distinguish between sentences with the same truth value, hence sentences are represented by truth values and logical terms are defined as functions on truth values. In (Lindström) 1st-order logic logical terms do not distinguish between objects (denotations and extensions of expressions) with the same mathematical (set theoretical) structure, hence objects are represented by mathematical (set theoretical) structures and logical terms are defined as functions on such structures. The semantic construction of Lindström logical terms is of course more complex than that of the logical terms of sentential logic, but that was to be expected.

To include logical terms of types $\langle 0,0\rangle$, $\langle 1\rangle$, and $\langle 2\rangle$ in a 1st-order Tarskian system, we adjust the syntax and the semantics in the following way:

4.3 Syntax

- If $R^{0\text{-}0}$ is a logical term of type $\langle 0,0\rangle$ and t_1, t_2 individual terms, then $\ulcorner R^{0\text{-}0}t_1t_2\urcorner$ is a wff.
- If Q^1 is a logical term of type $\langle 1\rangle$, Φ is a wff and x is a variable, then $\ulcorner(Q^1x)\Phi\urcorner$ is a wff.
- If Q^2 is a logical term of type $\langle 2\rangle$, Φ is a wff and x, y are variables, then $\ulcorner(Q^2xy)\Phi\urcorner$ is a wff.

4.4 Semantics

Relative to a universe A of cardinality α, a denotation function d, an assignment g and an indexing i of A by α,

- $v(\ulcorner R^{0\text{-}0}t_1t_2\urcorner[g]) = T$ iff $R_\alpha^{0\text{-}0}(i(\bar{g}(t_1)),i(\bar{g}(t_2))) = T$, where $i(\bar{g}(t_1))$, $i(\bar{g}(t_2))$ are the indices of $\bar{g}(t_1)$, $(\bar{g}(t_2)$, respectively.
  Informally: g satisfies $\ulcorner R^{0\text{-}0}t_1t_2\urcorner$ iff $R_\alpha^{0\text{-}0}$ assigns the value T to the pair of indices of the individuals assigned to t_1 and t_2 by g/d.
- $v(\ulcorner(Q^1x)\Phi\urcorner[g]) = T$ iff $Q_\alpha^1(I\{a \in A\colon v(\Phi[g(a/x)]) = T\}) = T$, where $I\{\ldots\}$ is the set of indices of all members of $\{\ldots\}$.
  Informally: g satisfies $\ulcorner(Q^1x)\Phi x\urcorner$ iff Q_α^1 assigns the value T to the set of indices of all $a \in A$ such that $g(a/x)$ satisfies $\ulcorner\Phi x\urcorner$.
- $v(\ulcorner(Q^2xy)\Phi\urcorner[g]) = T$ iff $Q_\alpha\ (I\{\langle a,b\rangle \in A \times A\colon v(\Phi[g(a/x)(b/y)]) = T\}) = T$.
  Informally: g satisfies $\ulcorner(Q^2xy)\Phi xy\urcorner$ iff Q_α^2 assigns the value T to the set of all pairs of indices of a, $b \in A$, such that $g(a/x)(b/y)$ satisfies $\ulcorner\Phi xy\urcorner$.

While Lindström's criterion does lead to a precise, informative definition of ɔgical terms (and truth), no rationale was offered for his criterion. True, his otion of logical term satisfies Mostowski's requirement that logical terms do ot distinguish the identity (individuality) of objects in a given universe, but ·hy should we take Mostowski's requirement as a criterion for *logicality*? A artial justification was given by Tarski who arrived at (essentially) the same riterion as Lindström. Taking his cue from Klein's program for classifying eometric disciplines according to the transformations of space under which ıeir concepts are invariant, Tarski suggested:

> ... suppose we continue the idea, and consider still wider classes of transformations. In the extreme case, we would consider the class of *all* one–one transformations of the space, or universe of discourse, or 'world', onto itself. What will be the science which deals with the notions invariant under this widest class of transformations? Here we will have very few notions, all of a very general character. I suggest that they are the logical notions, that we call a notion 'logical' if it is invariant under all possible one–one transformations of the world onto itself. (Tarski 1986/1966: 149)

3y way of justifying his proposal, Tarski said: "[This] suggestion perhaps ·ounds strange – the only way of seeing whether it is a reasonable suggestion s to discuss some of its consequences, to see what it leads to, what we have o believe in if we agree to use the term 'logical' in this sense." (Tarski 1986/1966: 149–50) And what this criterion commits us to believe in, Tarski went on :o say, is that no individual constant expresses a logical notion, that the only ɔinary logical relations of order 1 are the universal relation, the empty re-.ation, identity and diversity; that the only logical notions of order 2, type $\langle t_1 = 1, \ldots t_n = 1\rangle$ are cardinality notions; that all classical mathematical notions of order >1 are logical notions, etc. The totality of logical notions under this criterion coincides with the totality of terms definable by "purely logical means" in the system of *Principia Mathematica*, hence, Tarski concluded, the criterion stands "in agreement, if not with all prevailing usage of the term 'logical notion', at least with one usage which actually is encountered in practice." (Tarski 1986/1966: 145) Tarski, however, did not tell us why, or even whether, this particular usage should be preferred to others. Why should we not prefer a more restricted usage? a more liberal usage? an altogether different usage?

At least one philosopher, Etchemendy (1990), claims that Tarskian logic does not embed any rational criterion for logical terms, that, in fact, any term whatsoever can serve as a logical term in Tarskian logic. The inevitable consequence is that the very distinction between logical and non-logical truths (consequences) collapses. (Assume "is human" and "is mortal" are included in a Tarskian system as logical terms, based on the following semantic rules:

- $v(\ulcorner \text{Is human } (t)\urcorner[g]) = T$ iff $\bar{g}(t) \in \{a \in A\colon a \textit{ is human}\}$.
- $v(\ulcorner \text{Is mortal } (t)\urcorner[g]) = T$ iff $\bar{g}(t) \in \{a \in A\colon a \textit{ is mortal}\}$.

Then the intuitively logically indeterminate sentence "All humans are mortals" comes out logically true.)

Other questions regarding the distinction between logical and non-logical terms come from *linguistics*, especially the linguistic theory of generalized quantifiers.

5 The Linguistic Theory of Generalized Quantifiers

Linguists have found the logical theory of generalized quantifiers a fertile ground for applications. We can divide the surge of linguistic investigations of generalized quantifiers into two waves.[11] In the first wave linguistic determiners were correlated with 2-place cardinality quantifiers; in the second various linguistic constructions were analyzed as "polyadic" quantifiers. The first wave began with Barwise and Cooper (1981), Higginbotham and May (1981) and Keenan and Stavi (1986/1981). Influenced by Montague's analysis of noun phrases as 2nd-order entities on the one hand and by Mostowski's generalization of the logical quantifiers on the other, Barwise and Cooper suggested that NP's in general are generalized quantifiers. The logician's quantifiers correspond to the linguist's determiners, and the linguist's quantifiers are NP's. A quantified formula is obtained in two steps: First a determiner is attached to an open formula to form a quantifier, then the quantifier is attached to an open formula to form a quantified formula. The combination of "most" (a determiner) and "students" (an open formula, "x is a student") yields a quantifier, "most students," and the combination of "most students" and "passed the test" (an open formula) yields a sentence, "Most students passed the test," symbolized $\ulcorner$Most (students) $\hat{x}$[Passed the test (x)]$\urcorner$. What is, for the logician, a 2-place quantification of the form $\ulcorner Q^{1\text{-}1}x(\Phi x, \Psi x)\urcorner$ is, for the linguist, a 1-place quantification of the form $\ulcorner D(\Phi)\,\hat{x}[\Psi x]\urcorner$. Semantically, determiners are defined as functions from sets to quantifiers (construed as sets of sets). E.g. *Most*($\{x$: x is a student$\}$) = *Most–students* (the set of all sets which include most students as members). Quantifiers are defined as functions from sets to truth values. E.g. *Most–students*($\{x$: x passed the test$\}$) = T/F. (*Most–students*(X) = T iff $X \in$ *Most–students*.) Some determiners are logical, others are non-logical. Determiners expressing the standard logical notions (e.g. "all," "some," "no," "neither," "at least/at most/exactly 2") are logical; determiners expressing other notions (e.g. "most," "many," "few," "a few," "all but John") are non-logical.[12] Quantifiers are, in general, non-logical: "most students," "most teachers," "all children," "all elephants" are distinct non-logical quantifiers. Since NP's in general are quantifiers, proper names are also quantifiers: "Mercury is a planet" has the form $\ulcorner$Mercury $\hat{x}$[x is a planet]$\urcorner$. This sentence comes out true iff the quantifier "Mercury" (the set of all sets which include Mercury as a member) assigns the value T to the set of all planets. The account also accommodates quantifications with a single open formula. In "Something is blue," for example, "Some" is a determiner, "thing" is an open formula and "Some(thing)" is a quantifier. The sentence is symbolized: ⌜Some(thing) $\hat{x}$[Blue(x)]⌝.

The only rule that applies to quantifiers (non-logical entities) as "part of the logic," according to Barwise and Cooper, is *conservativity* (or the "living on" condition). We can express this rule in terms of the linguist's quantifiers by: $D(B)[C] = T$ iff $D(B)[B \cap C] = T$, and in terms of the logician's quantifiers by: $Q(B,C) = T$ iff $Q(B,B \cap C) = T$. Informally, conservativity says that the first set (the extension of the left open formula) in a linguistic quantification determines the relevant universe of discourse. "All," "some," "most," are conservative ("Most students passed the test" is equivalent to "Most students are students who passed the test"), while "only" and "There are more . . . than ---" are not conservative ("Only women are allowed in the club" is not equivalent to "Only women are women who are allowed in the club").

Whereas Barwise and Cooper accepted only determiners defined in terms of the standard logical quantifiers as logical, other linguists and logicians (Keenan & Stavi 1986/1981, van Benthem 1983, Westerståhl 1985) took Lindström's criterion as a criterion for logical determiners, viewing invariance under isomorphic structures as expressing the idea that logical terms are topic neutral (van Benthem and Westerståhl). But, according to some of these researchers, various conditions restrict the scope of logical (and non-logical) determiners. Among these are, in addition to *conservativity*, (i) *Continuity* (van Benthem, 1983): a determiner D assigns the value T "continuously," i.e. if $C_1 \subseteq C \subseteq C_2$ and $D(B)[C_1] = D(B)[C_2]$, then $D(B)[C_1] = D(B)[C] = D(B)[C_2]$. This is a particular case of *graduality*: "a determiner should not change its mind too often." (457) (ii) *Constancy* (Westerståhl 1985): if $A \subseteq A'$ and D is a determiner, then D_A and $D_{A'}$ coincide over A. (iii) *Uniformity* (van Benthem, 1983): "the behaviour of D should be regular ("the same") across all universes" (457), where "regular (is the same)" is open to a "hierarchy" of interpretations. None of these requirements is satisfied by all binary Mostowskian quantifiers: "Only" is not conservative, "An even number of" is not continuous, "Half the objects in the universe are both . . . and ---" is not constant, and the quantifier Q defined by: $Q(\alpha) = Most_\alpha$ if α is even and Few_α if α is odd or infinite, is not uniform.

The second wave of linguistic applications of generalized logic centered around polyadic quantifiers, quantifiers binding a formula, or a finite sequence of formulas, by means of two or more variables. (See, among others, Higginbotham & May 1981, Keenan 1987b, May 1989, van Benthem 1989, Sher 1991.) Already in 1981 Higginbotham and May suggested that polyadic quantifiers will help us solve nagging problems of cross reference and "bijective" *wh*-questions. Thus by positing a polyadic, "All–Some" quantifier in two variables, we can explain the anaphoric relations in cross reference (Bach–Peters) sentences like "Every pilot who shot at it hit some Mig that chased him." Polyadic analysis can also explain how questions with multiple singular *wh*-phrases have a bijective interpretation, e.g. how "Which man saw which woman?" allows the bijective answer: "John saw Jane, and Ron saw Nancy."

Other examples of polyadic constructions in natural language are "Different students answered different questions on the exam" (Keenan 1987b), "No one loves no one" (May 1989), "For every drop of rain that falls, a flower grows" (Boolos 1981)[13], etc. Branching quantifiers, or non-linearly ordered quantifier prefixes, can also be regarded as polyadic quantifiers (Van Benthem 1989). Examples of statements analyzed in the literature as branching quantifications are "Some relative of each villager and some relative of each townsman hate each other" (Hintikka 1973b), "Quite a few boys in my class and most girls in your class have all dated each other" (Barwise 1979), "Most of my friends have applied to the same few graduate programs" and "Most of my right hand gloves and most of my left hand gloves match (one to one)" (Sher 1990). What is distinctive of these sentences is the occurrence of two or more Mostowskian quantifiers neither one of which is in the scope of the other.

None of the conditions mentioned above has survived the passage from determiners to polyadic quantifiers: the only constraint on polyadic quantifiers is logicality, construed as invariance. But the nature of invariance ("Invariance under what?") is an open question. Some have interpreted invariance in Lindström's sense (Keenan 1987b, van Benthem 1989), but others have questioned the type of invariance involved. Thus Higginbotham and May (1981) asked whether for binary polyadic quantifiers the condition is invariance under isomorphisms based on permutations of individuals (Lindström), permutations of pairs of individuals, permutations of pairs of individuals with a distinguished first or second element, etc.

Mostowski, Lindström, Tarski, Etchemendy, linguists studying natural language quantifiers, and others (Tharp 1975, Peacocke 1976, Hacking 1979, McCarthy 1981, etc.) have come up with an array of views (and points of view) regarding the existence and content of criteria for logicality. One way to approach the issue is to predicate it on a more fundamental question, for example (Tharp 1975): "What is the task (the point, the intended contribution) of logic?" If we can identify a central task of logic and determine what role logical terms play in carrying out this task, we will be able to view the distinction between logical and non-logical terms as a distinction between terms that can and terms that cannot fill this role, or terms that will and terms that will not contribute to the logical project by "acting" as logical terms. Below, I will present my own solution to the question of logicality based, in part, on Sher 1991. Although my general approach is influenced by Tharp, my analysis and subsequent solution are very different from his.

6 A Philosophical Basis for the Distinction Between Logical and Non-Logical Terms

The primary task of logic is often conceived of as the development of a method for identifying logical consequences, logical truths being a particular case. But given this conception, the question immediately arises: "What kind of consequences are logical?" Many say that logical consequences are due to the structure of sentences, where structure is a function of the specific logical terms present and their arrangement. Thus, Quine (1970: 48) says: "Logical implication [consequence] rests wholly on how the truth functions, quantifiers, and variables stack up. It rests wholly on what we may call, in a word, the logical structure of the . . . sentences [involved]." This characterization, however, gives rise to the question: "Which terms are logical? (What is the *logical* structure of a given sentence?)" So, from the point of view of our present inquiry (into the notion of logical term) this approach is unhelpful. What we need, in order to demarcate the logical terms, is an intuitive characterization of logical consequence that is independent of this demarcation.

A characterization of logical consequence satisfying this requirement was given by Tarski in "On the Concept of Logical Consequence" (1936a). According to Tarski (as I read him), a consequence is logical iff it satisfies two fundamental conditions: (a) *necessity*, and (b) *formality*. In Tarski's words: "Certain considerations of an intuitive nature will form our starting point. Consider any class K of sentences and a sentence X which follows from the sentences of this class. From an intuitive standpoint it *can never happen* that both the class K consists only of true sentences and the sentence X is false. Moreover, . . . we are concerned here with the concept of logical, i.e. *formal*, consequence. . . ." (Tarski, 1936a: 414)[14] In some accounts of logical consequence necessity, but not formality, is mentioned as a primary trait (see Etchemendy, 1990, and references there). But these accounts, Tarski tells us, miss the distinctive feature of *logical*, as opposed to other types of consequence. Logical consequence satisfies a more stringent condition than mere necessity: logical consequence is *formal and necessary* (formally necessary).

Taking Tarski's characterization as my starting point, I would first like to say a few words about formality. It is common to view formality as a syntactic condition: the formality of logical consequence is captured by the various syntactic definitions, while its necessity is captured by the semantic definition. I believe this approach is wrongheaded. The intuitive notion of logical consequence includes the idea that logical consequence is necessary in a special way, namely, in a *formal* way. To capture the intended notion of logical consequence, the semantic account cannot disregard the formal nature of intuitively logical consequences. Two examples of intuitively necessary but non-formal consequences are: (i) "The ball is all blue; therefore, it is not yellow," and (ii) "John is a bachelor; therefore he is unmarried." These inferences clearly rest on non-formal principles – metaphysico-physical principles in the first case and lexical conventions in the second – and this fact can be used to explain why we tend not to view them as strictly logical.

Tarski examined two syntactic definitions of logical consequence and found them lacking: the standard proof theoretical definition, and a substitutional definition. I will begin with the proof theoretical definition. In modern terminology we can formulate this definition as follows: Given a standard 1st-, or higher-order, logic $\mathscr{L}$ with a set of axioms, $\mathscr{A}$, and a set of rules of proof, $\mathscr{R}$: if Γ is a set of sentences of $\mathscr{L}$ and σ is a sentence of $\mathscr{L}$, then σ is a logical consequence of Γ iff there is a (finite) proof of σ from Γ (using the axioms in $\mathscr{A}$ and the rules in $\mathscr{R}$). Now, consider the inference "P(0), P(1), . . . , P(n), . . . ; therefore all natural numbers have the property P," expressed either in 1st- or in higher-order logic. This inference is intuitively formal and necessary, yet it cannot be established by means of the standard proof method. Furthermore, it follows from Gödel's incompleteness theorem that not even by adding (reasonable) rules of proof can we establish all intuitively formal and necessary consequences.

The second syntactic definition of logical consequence is substitutional. This definition says that given a natural (interpreted) language L, σ is a logical consequence of Γ iff there is no uniform substitution for the primitive non-logical constants of L (by grammatically compatible primitive terms of L) under which all the sentences of Γ come out true and σ comes out false. There are (at least) three problems with this definition: the first was pointed out by Tarski (1936a) and the other two are drawn from an analysis of Etchemendy (1990).[15]

(i) The substitutional definition is exceedingly sensitive to the richness of the non-logical lexicon: if L has a limited non-logical vocabulary, then certain consequences which fail the intuitive test pass the substitutional test. For example, let "Tarski," "Gödel," "is a logician" and "is a male" be the only non-logical constants of L. The intuitively incorrect consequence "Tarski is a male; therefore Tarski is a logician" comes out logically valid in L.

(ii) The substitutional method does not have the resources for distinguishing between logical and non-logical terms. Its only resources are grammar, lexicon and the notion of preservation of material truth under substitutions, and these do not suffice to decide the logical status of a given term. The substitutional method, therefore, reduces the distinction between logical and non-logical terms to an arbitrary, or conventional, distinction between "fixed" and "non-fixed" terms, and the notion of logical consequence is relativized to arbitrary divisions of terms into fixed and non-fixed.[16] It can easily be seen that relative to some selections of fixed terms, the "wrong" consequences pass the substitutional test, while relative to others, the "right" consequences fail. Thus, if "Tarski" and "is a logician" are among the fixed terms, "Tarski is a male; therefore Tarski is a logician" passes the substitutional test (and this happens no matter how rich the lexicon of L is). And if "(" and ")" are the only fixed terms, then, assuming the language is modestly rich, "$P\&(Q\&R)$; therefore $(P\&Q)\&R$" fails the substitutional test.

(iii) The substitutional method takes into account only facts about the *actual* world. According to the substitutional definition a sentence is logically true iff all its substitutional variants are true – true simpliciter, i.e. true in the actual world. Thus, from the point of view of the substitutional theory logical truth is *actual* truth preserved under variations in language (non-fixed constants). It follows that if a non-formal, non-necessary truth is insensitive to the non-fixed constants of the language, in particular, if it does not contain non-fixed terms, it is automatically judged to be logically true. This problem arises even on the standard selection of logical terms: "There are at least two objects" is expressible by purely logical vocabulary ($\ulcorner\exists x\exists y(x \neq y)\urcorner$), hence its truth is equated with its logical truth. This sentence is a paradigm of logical indeterminacy, but having no substitutional instances other than itself, it comes out logically true. The same distortion occurs in the case of logical consequence: Assuming the standard selection of logical terms, "There are exactly two objects; therefore X" passes the substitutional test for any sentence X.[15]

Tarski rejected the proof-theoretical and substitutional definitions and decided to use a different method. The "semantic" method, developed in Tarski (1933) naturally suggested itself to him. Below I will present my own version of Tarski's theory. I will show how the considerations motivating this theory provide a philosophical foundation for Lindström's criterion, and how, given this criterion, the theory avoids the limitations and pitfalls of the proof-theoretical and substitutional definitions.

What is a semantic theory? Following Tarski I view semantic theories as theories that deal with concepts relating language to the world (in a broad sense): "We shall understand by semantics the totality of considerations concerning those concepts which, roughly speaking, express certain connexions between the expressions of a language and the objects and states of affairs referred to by these expressions." (Tarski 1936b: 401) There are two types of semantic concepts: those that satisfy this characterization directly and those that satisfy it indirectly. "Reference" and "satisfaction" fall under the first category, "truth" and "logical consequence" under the second. Reference is a relation between a term and an object it refers to, and satisfaction is a relation between a formula and an object (a sequence of objects, a function from the variables of the language to objects in the universe) satisfying it. Truth and logical consequence, however, are linguistic properties (relations): truth is a property of sentences, and logical consequence is a relation between a sentence and a set of sentences. Why do we view them as semantic? One common answer is that truth and logical consequence are semantic because they are definable in terms of semantic relations (reference and satisfaction). I think this answer puts the cart before the horse. Truth is definable in terms of reference and satisfaction because it has to do with objects and their relations to language. Truth holds or does not hold of a given sentence $\mathfrak{s}$ iff the objects referred to in $\mathfrak{s}$ possess the properties (stand in the relations) attributed to them by $\mathfrak{s}$. More generally, a linguistic property is semantic iff it holds (or fails to hold) of a given linguistic entity ℓ due to certain facts about the objects referred to in ℓ; and similarly for relations. To view logical consequence as a semantic relation is, thus, to view it as a relation between linguistic entities, based on a relation between the objects referred to by these entities. Semantics reduces statements about language to statements about objects. I will not be able to discuss the reduction of "truth" here,[16] but in the case of "logical consequence" semantics reduces "The sentence σ is a logical consequence of the set of sentences Γ" to something like "The properties attributed to objects by σ stand in the (objectual) relation $\mathfrak{R}$ to the properties attributed to objects by Γ." To understand the nature of logical consequence as a semantic relation is thus to understand (i) the nature of reference, and (ii) the nature of the objectual relation $\mathfrak{R}$. Leaving reference aside, we can view the intuitive conditions on logical consequence – necessity and formality – as conditions on $\mathfrak{R}$. Necessity requires that $\mathfrak{R}$ hold *necessarily;* formality – that $\mathfrak{R}$ take into account only *formal* features of the objects and properties (relations) involved. What is the objectual relation $\mathfrak{R}$?

Consider the intuitively logically valid inference, (1): "Something is white and tasty; therefore, something is tasty." Why is (1) logically valid? Adopting Mostowski's way of viewing quantifiers, we can say that (1) is logically valid because (i) its premise says that the intersection of the sets of white and tasty things is not empty, (ii) its conclusion says that one of the intersected sets, namely the set of tasty things, is not empty, and (iii) *whenever* an intersection of sets is not empty, each of the intersected sets is not empty. Looked at in this way, semantics reduces "σ is a logical consequence of Γ" to "$f(\sigma)$ $\mathfrak{R}$ $f(\Gamma)$," where f extracts the relevant content of σ and Γ (in our example: a set is not empty [$f(\sigma]$, its intersection with another set is not empty [$f(\Gamma)$]), and $\mathfrak{R}$ is an objectual expression of generality, roughly "whenever."

In *Tractatus Logico–Philosophicus*, 6.1231–6.1232, Wittgenstein seemingly rejected this way of looking at logical consequence. Speaking in terms of "logical truth" ("logical proposition", in his terminology), Wittgenstein said: "The mark of a logical proposition is *not* general validity. To be general means no more than to be accidentally valid for all things." This observation led Wittgenstein to object to the reduction of logic to generality, and a similar objection was made recently by Etchemendy (1990).[19] But Wittgenstein's problem does not arise on our analysis: logical consequence is not reducible to just any kind of generality, logical consequence is reducible to a special kind of generality, to generality satisfying the intuitive constraints of necessity and formality. Necessity and formality constrain both the function f and the scope of "*whenever*". Formality requires that logical consequence depend only on formal features of the objects involved (non-emptiness of sets and intersections), not their material features (whiteness and tastiness of things), and necessity requires that "whenever" not be restricted to any particular universes, but range over all possible universes. Combining necessity and formality, we can say that logical consequence is reducible to *formal generality*. It is a formally universal fact (a fact that holds in all formally possible structures of objects) that if an intersection of sets is not empty, each of the intersected sets is not empty, and it is due to this formal and necessary fact that (1) is logically, i.e. formally and necessarily, valid.

Tarskian semantics systematizes the reduction of logical truth and logical consequence to formal universality. What is the role of logical terms in this reduction? Consider (1) above. The formal relation underlying (1) is not affected by changes in the extensions of "white" and "tasty," but changes in the extension of "something" and "and" may very well affect it. We can explain the difference between the two pairs of terms by the fact that the formal relationship underlying (1) holds no matter what (formally possible) sets of individuals "white" and "tasty" denote: Given any formally possible universe of individuals and any two sets of individuals in this universe (sets of white and tasty things, sets of black and sour things, etc.), if the intersection of the two sets is non-empty, so is each set non-empty. But the same does not hold for unions: It is formally possible for a union of two sets to be non-empty while one of the sets is empty. Similarly, it is formally possible for an intersection of two sets to be empty without either set being empty. This is the reason the validity of (1) depends on "and" and "something." If we replace the denotation of "and" by the denotation of "or" or the denotation of "something" by the denotation of "nothing," (1) will turn logically invalid. "And" (in $\ulcorner \ldots x \ldots$ and --- x ---$\urcorner$) denotes an intersection, "something" denotes the property of being non-empty, and the laws governing the intersection and non-emptiness of sets hold in all formally possible structures.

The role of logical terms in Tarskian logic is, thus, to mark formal features and structures of objects, the kind of features and structures responsible for logical consequences. Since logical terms have denotations in all formally possible structures of objects, logical terms are "universal" terms, terms denoting properties applicable to all formally possible objects: humans, dogs, cells, atoms, colors, natural numbers, real numbers, etc. The standard logical terms satisfy this requirement, hence consequences based on these terms are genuinely logical. But the question naturally arises whether all formally necessary consequences are based on the standard logical terms. Consider the inferences (1) "There is exactly one human; therefore, there are finitely many humans," and (2) "There is exactly one human; therefore there are at most ten humans." In standard 1st-order logic (2) is considered logically valid while (1) is logically indeterminate. But is (1) less formally necessary than (2)? Is it intuitively more possible for a singleton set to be infinite than to include ten elements and above?

The task of Tarskian semantics, as I understand it, is to provide a "complete" system for detecting logical, i.e. formal and necessary, consequences. One way of achieving this result is by turning to standard higher-order logic, but another way is to extend standard 1st-order logic by adding new logical terms (e.g. the quantifier "there are finitely many x"). How do we determine the totality of logical terms? To determine the totality of logical terms is to determine the totality of formal (universal) structures of objects. Each formal structure is the extension of some logical term, and each logical term denotes a formal structure. To identify formal structures we will use the best universal theory of formal structure available (a theory applicable to structures of any kind of individuals). Currently ZF (with urelements) or one of its variants appears to be a reasonable choice. Based on this theory we will develop a criterion for formal identity of structures, and based on this criterion we will say that a term is logical iff it does not distinguish between formally identical structures. But this is exactly what Lindström's criterion says, based on the model-theoretic notion of structure. A term is logical iff it is invariant under isomorphic structures. I.e. a term is logical iff it does not distinguish isomorphic extensions of its arguments. Based on this criterion, all cardinality quantifiers are logical, the 2nd-order property of being a symmetric (transitive, reflexive) relation is logical, etc. More generally, any mathematical term definable as a higher-order term is essentially logical. Take, for example, the term "two." As an individual term "two" denotes a particular individual – the number two – hence it fails the logicality test, but as a higher-order term "2" denotes a formal structure – the set of all sets isomorphic to {0,1} – hence is a bona fide logical term. (This comparison explains why individual constants are not included in (L): individual constants denote atomic objects, objects with no structure, formal or non-formal.)

The characterization of logical terms as universal and formal allows us to explain how Tarskian semantics avoids one of the stumbling blocks of substitutional semantics, namely, the relativity of its notion of consequence to arbitrary selections of fixed terms: Lindström's criterion precludes the use of any non-formal or non-universal terms ("Tarski," "is a logician," etc.) as logical (fixed) terms, ruling out inferences obtained by holding such terms fixed. The two remaining problems are circumvented by the introduction of Tarskian models and their specific features. Models, in Tarskian semantics, represent formally possible structures of objects, and the notion of truth in all models (truth in all models of Γ) is not dependent either upon the size of the non-logical vocabulary or upon the size (and other contingent traits) of the actual world: Whether "Tarski" is the only singular term of L or not, "Tarski" is assigned a great many distinct individuals in different models for L, enough to establish a counter-example to "Tarski is X", for any non-logical primitive predicate X. Likewise, truth in a Tarskian model is not truth in the actual world, but truth in a formally possible structure, and the notion of formally possible structure is not constricted by contingent facts (e.g. the cardinality of the actual universe).[20]

7 The Logical Nature of Tarskian Semantics

The third question I raised in this paper was: Is Tarskian semantics specifically designed to accommodate the needs of logic, or can the same apparatus be used to explain other aspects of the relation between language and the world? A complete answer to this question is beyond the scope of the present paper, but I would like to point out two ways in which Tarskian semantics is inherently logical: (a) its choice of "fixed" terms, (b) its method of representing objects and states of affairs. The first point should be obvious by now: Tarskian semantics, even Tarski's general definition of truth (Tarski 1933), is restricted to languages whose "fixed" terms are *logical*, and the recursive definition of truth via satisfaction reflects this fact. This definition employs "fixed" functions which correspond to the logical terms of the language, and these functions take into account only formal features of objects in their domain.

The second point has to do with the way objects and states of affairs are represented in Tarskian semantics, i.e. with its apparatus of models. Briefly, we can present this point as follows: As a logical semantics, Tarskian semantics is interested in features of objects and states of affairs that contribute to logical consequences and only those. That is to say, Tarskian semantics is concerned with (universal) formal features of objects and states of affairs and nothing else. Tarskian models disregard the diversity of objects and the multitude of non-formal properties they possess (non-formal relations in which they stand). All objects, physical and mental, abstract and concrete, microscopic and macroscopic, fictional and real, are represented as members of a set theoretical entity, the "universe" of a model, i.e. a set. All properties of these diverse objects are represented as sets of members of the universe, all relations are represented as sets of n-tuples of members of the universe, etc. Nothing is possibly both dead and alive, but in some Tarskian models things are. Tarskian semantics respects the exclusionary relation between "Exactly two things are X" and "Exactly three things are X" since this relation is formal, but it does not respect the exclusionary relation between "x is dead" and "x is alive" since this relation is not formal. Similarly, Tarskian semantics reduces the multitude of ways in which objects possess properties and stand in relations to a single formal relation, set membership, in spite of the fact that an object possesses, say, a moral property (e.g. being virtuous), or a propositional attitude property (e.g. wishing to be president) in a largely different way from that in which it possesses a physical property (e.g. occupying spatio-temporal region $xyzw$). But Tarskian semantics is not interested in these differences. Set membership is adequate for possession of formal properties, and for the purposes of Tarskian semantics, there is no need for something more elaborate. From the point of view of Tarskian semantics, only the formal skeleton of "object x possesses property P" is relevant.

With this my discussion of semantics and logic has come to an end. I have asked three questions in this paper: (A) Is there a philosophical basis for the distinction between logical and non-logical terms? (B) Can we develop a constructive definition of logical terms based on (A)? (C) To what extent is modern semantics tied up with logic? I proposed an outline of a positive answer to (A) and (B) and some considerations pertaining to (C). Modern semantics sprang out of a distinctly logical conception of semantics. This conception originated in Tarski's theories of truth and logical consequence in the 1930s and has recently been extended following the generalization of logical terms by Mostowski and others. I have shown that the line drawn by Lindström (and the later Tarski) between logical and non-logical terms is philosophically sound, and I have given an outline of a constructive definition of logical terms modeled after the Boolean definition of the logical connectives. As for the many branches and developments in modern semantics, to the extent that these are an outgrowth of Tarskian semantics they are rooted in a *logical* theory of the relation between language and the world. How far semantics can and should go beyond its logical roots is left an open question.

NOTES

1 See Tarski (1933) and (1936a). The inductive definition of syntax is not original with Tarski (note Hilbert and Ackermann 1928), but its central place in semantics is due to Tarski.

2 See, among others, Barwise and Feferman (1985), Hughes and Cresswell (1968), Hintikka (1969), Davidson (1984), Montague (1974), May (1985) and many of the references below.

3 See Enderton (1972), Section 1.2.

4 For an example, see Keisler (1970).

5 Lindström's criterion is slightly different from (*L*): (i) it includes connectives, (ii) it does not apply to all predicates of orders 1–2. More specifically, it does not apply to relations involving individuals (e.g. the relation $\mathfrak{R}$ of type ⟨2,0⟩, where $\langle R, a\rangle \in \mathfrak{R}$ iff R is an ordering relation with a smallest element and *a* is its smallest element.) Tarski's criterion does apply to all predicates, but he uses a slightly different conceptual scheme.

6 Lindström's symbolization is slightly more complicated. He would write "$Qxy(\Phi x, \Psi y)$" where I write "$Qx(\Phi x, \Psi x)$".

7 I use "$Q^{1\text{-}1}$ B's are C's" to represent the general case of a predication of type 1-1 although some predications of this type (e.g. "There are more B's than C's") would be more naturally represented by a different locution.

8 Unlike $Q^{1\text{-}1}_a$, $Q^{1\text{-}1}$ is a "class" function rather than a "set" function, i.e. a *class*, rather than a <u>set</u> of pairs. In what follows I use the term "function" for both. It should be clear from the context whether "set" or "class" is meant.

9 Here I treat cardinals as sets of ordinals, but ordinals have to be treated as atomic elements: $\{1\}$ and $\{\{0\}\}$, for example, represent different extensions, indeed, extensions of terms of different types.

10 It follows from this step that logical terms may "mean" different things in universes of different cardinalities. Thus, a function, $\exists_F\forall_I$, that assigns to all finite universes A the quantifier $\exists_A$ and to all infinite universes B the quantifier $\forall_B$ is a logical term. This may sound strange at first, but we must remember that even if $C = D$, $\langle A, \langle C\rangle\rangle$ and $\langle B, \langle D\rangle\rangle$ are structurally different, and so various logical quantifiers distinguish between them. In fact, many "natural" quantifiers are sensitive to the size of the underlying universe. Thus "most^1" assigns the value T to $\{a,b,c,d\}$ in a universe with 6 individuals but not in a universe with 60,000 individuals. Ratio quantifiers can be construed as exploiting the fact that quantifiers may vary according to the size of the given universe: "1/2" is "exactly one" in a universe with two elements, "exactly two" in a universe with four elements, etc. In constructing specific logical apparati various considerations beyond logicality are, of course, taken into account: we may wish to restrict ourselves to "natural" logical terms, logical terms that are useful for a particular purpose, etc. Clearly, many factors combine to determine the logical apparatus of natural language.

11 This division is both analytical and historical, although historically, there are a few exceptions (e.g. Higginbotham and May, 1981).

12 It should be emphasized that I am referring to Barwise and Cooper's claim that these determiners are non-logical in the sense that they do not denote unary quantifiers of first-order logic. Van Benthem (1986) and (1989), Westerståhl (1989), and Keenan (this volume) characterize logical determiners as those which denote functions that are permutation invariant for isomorphic structures on the given universe. On the latter view of logicality, "most", for example, can be taken to denote a logical binary determiner relation. See Lappin (1995b) for a discussion of generalized quantifiers and logicality.

13 Boolos identified statements of this kind as "non-firstorderizable" (relative to the *standard* 1st-order system). For a polyadic analysis of such statements, see Sher (1991: 103).

14 The first underline is mine.

15 See Sher (forthcoming).

16 Compare with Quine's analysis of substitutional quantification. In substitutional quantification, according to Quine, the choice of substitution classes is arbitrary: "substitutional quantification makes good sense, explicable in terms of truth and substitution, no matter what substitution class we take – even that whose sole member is the left-hand parenthesis." (1969: 106)

17 The notion "actual world" is ambiguous, but all we need here is the fact that the concept of truth simpliciter involves a more restricted notion of "world" than the concept of formal and necessary truth.

18 I am currently working on a paper on Tarski's theory of truth. My account is revisionist in the sense that it is concerned with what Tarski's theory actually does, not with what "the historical" Tarski said it does.

19 The connection between Etchemendy and Wittgenstein was pointed by Garcia-Carpintero (1993).

20 My reasons for regarding formal terms as logical are primarily philosophical. The question whether linguistically (empirically), too, formal terms should be regarded as logical requires a separate investigation. May (1991: 353) gives a preliminary positive answer based on the difference between the way a child acquires formal terms and the way s/he acquires non-formal terms. The promising work on natural language polyadic quantifiers (i.e. quantifiers based on (*L*)) also yields support to the "formal" view of logical terms. The difference between philosophical and linguistic conceptions of logicality is discussed in Lappin (1991).

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