

# PHIL124 PHILOSOPHY OF MATHEMATICS

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Professor: Paolo Santorio

UC San Diego, Spring 2019

Office Hours: Th 3:30-5:30, H&SS 8004

TuTh 2-3:20, H&SS 1315

We are all taught arithmetic in early age, and simple arithmetical statements like  $7+5 = 12$  seem as banal as anything can be. But what are statements like this about, and how can we come to know them? Questions of this sort have taken center stage in philosophy since philosophers became concerned with epistemology and the limits of human knowledge.

This course focuses on discussions of the metaphysics and epistemology of mathematics in contemporary analytical philosophy (i.e. post-1879). It is divided into two main units. In the first, we will survey two beginning-of-century major views of mathematical epistemology and mathematical practice, i.e. logicism and intuitionism. In the second, we will focus on the debate on the ontology and epistemology of mathematics started by philosopher W. v. O. Quine in the 1960s.

## READINGS AND TEXTBOOK

The textbook for the class is:

Stewart Shapiro, *Thinking about Mathematics*, Oxford University Press, 2000

We won't read it cover to cover, but we'll use about half of it. We will also be reading large chunks of the following book:

Gottlob Frege, *The Foundations of Arithmetic*. (Any edition/translation is fine.)

Copies of Frege's *Foundations of Arithmetic* can be found online, though if you are interested in reading philosophy beyond college coursework I recommend buying a copy; it is a truly amazing piece of work.

In addition, I will assign about 12-15 research articles, all of which will be posted on TritonEd.

## ASSIGNMENTS

You will be asked to hand in eight weekly take-home quizzes, write one argument analysis and two papers. An argument analysis is a short assignment (700-900 words) where you are going to be asked to reconstruct an argument in a passage and critically evaluate it. In addition, I expect you to come to class having done the readings and participate in discussion. The breakdown of points is as follows:

- Short quizzes (best six out of seven): 15%
- Class participation: 10%
- Argument analysis: 15%
- Paper 1: 30%
- Paper 2: 30%

**Take-home quizzes.** On seven weeks within the term, you will be asked to fill in a short take-home quiz. The quiz will consist of some multiple choice questions and some questions requiring a short written answer. It should only take 20-30 minutes (or less) to fill in. The bar for this assignment won't be high, and I expect that most quizzes that are handed in will get a full score.

*The quizzes may be handed in exclusively in class. I will not accept quizzes submitted by email.*

(I may make exceptions for special circumstances, possibly requiring documentation.)

I will read each of your quizzes, but I will only count the grades of the best six. This means also that you may simply skip one quiz with no consequence to your grade. Decide to skip a quiz at your own risk, especially at the beginning of the semester!

**Argument analysis and papers.** Topics for papers will be posted at least two weeks before the deadline. The argument analysis will be required to be between 700 and 900 words. The two papers will be required to be between 1,400 and 1,700 words.

#### EXTENSION POLICY

I will use a 'time bank' method for granting extensions for papers. Each of you starts the semester with an allowance of 48 hours of extension time in the 'bank', divided into blocks of 12 hours. You may use your allowance at any point, no questions asked. The only constraint is that **you are allowed to use at most 24 hours of extension for the argument analysis.**

Please just flag to me that you intend to use one or more blocks when you do. Once you run out of your allowance, I will not grant further extensions. (I may make exceptions for special circumstances, possibly requiring documentation.)

After your extension allowance runs out, your grade will be reduced 1/3 of a mark for each day a paper is late. For example, an A- will be reduced to a B+ in one day, a B in two days, and a B in three days.

The foregoing exclusively applies to the argument analysis and the papers. There will be no extensions on quizzes.

#### LAPTOP USAGE POLICY

Use of laptops, tablets, and phones is not allowed in class, except in special cases. Contact me for permission if you have specific reasons why you need to use any of these devices.

#### PLAGIARISM

Please familiarize yourself with university policies on plagiarism, cheating, and academic integrity.

In general, plagiarism is presenting as yours work that you did not produce. This can happen in a variety of ways. Note in particular that you can plagiarize a text even if you paraphrase or change some of the words in it. Note also that plagiarism need not be knowing or intentional. Always err on the side of citing any sources that have influenced your thinking, including websites and people you have had conversations with.

Any form of cheating or plagiarism will be reported immediately. Penalties for academic integrity infractions may include failing the assignment, failing the course, suspension, and expulsion.

## WEEK-BY-WEEK SCHEDULE (TENTATIVE)

*Notice: schedule and readings may be subject to change. Please make sure you are using the latest version.*

### Week 1: Introduction; historical preliminaries

**Lecture 1.** (April 2nd) Introduction. Philosophy of mathematics and its place in philosophy

*No required reading.*

**Lecture 2.** (April 4th) Historical preliminaries: Kant and Mill

*Required reading:* Shapiro *TaM*, chapter 4; Kant, excerpt from *Prolegomena*, pages 13–49.

*Additional reading:* J. A. Coffa, chapter 1 of *The Semantic Tradition from Kant to Carnap*.

### Week 2: Logicism

**Lecture 3.** (April 9th) Frege's logicist program, 1/3

*Required reading:* Shapiro *TaM*, chapter 5; Frege, *Foundations of Arithmetic* §§1–25, 45–54.

**Lecture 4.** (April 11th) Frege's logicist program, 2/3

**Quiz 1 due in class**

*Required reading:* Frege, *Foundations of Arithmetic* §§55–71.

### Week 3: Logicism, continued

**Lecture 5.** (April 16th) Frege's logicist program, 3/3

*Required reading:* Frege, *Foundations of Arithmetic*, §§72–91; Heck, "Frege's Theorem: An Introduction".

**No class.** (April 18th) No class

**Argument analysis due (online) on April 20th, 11:59pm**

### Week 4: Intuitionism

**Lecture 6.** (April 23rd) Intuitionism: the mathematical subject

*Required reading:* Shapiro, *TaM*, chapter 7; Heyting, TBA.

**Lecture 7.** (April 25th) Intuitionism: from language to mathematics

**Quiz 2 due in class**

*Required reading:* Dummett, "The Philosophical Basis of Intuitionistic Logic".

### Week 5: Realism and Indispensability

**Lecture 8.** (April 30th) Two kinds of realism: Gödel and Quine

*Required reading:* Maddy, selections from *Naturalism in Mathematics*.

**Lecture 9.** (May 2nd) Quine and Putnam on Indispensability

**Quiz 3 due in class**

*Required reading:* Putnam, “What is Mathematical Truth?”

**Paper 1 due (online) on May 5th, 11:59pm**

### **Week 6: The Benacerraf Problem; perceptual realism**

**Lecture 10.** (May 7th) Benacerraf on Mathematical Truth

*Required readings:* Benacerraf, “Mathematical Truth”.

**Lecture 11.** (May 9th) Maddy’s perceptual realism

**Quiz 4 due in class**

*Required reading:* Maddy, excerpts from “Realism in Mathematics”.

### **Week 7: Responses to the Benacerraf problem: fictionalism**

**Lecture 12.** (May 14th) Fictionalism: Overview

*Required reading:* Shapiro, *TaM*, chapter 9.

**Lecture 13.** (May 16th) Yablo’s fictionalism

*Required reading:* Yablo, “Go Figure: A Path through Fictionalism”.

### **Week 8: Responses to the Benacerraf problem: structuralism**

**Lecture 14.** (May 21st) Introduction to Structuralism

*Required reading:* Shapiro, *TaM*, chapter 10; Benacerraf, “What Numbers Could not Be”.

**Lecture 15.** (May 23rd) Structuralism, continued

**Quiz 5 due in class**

*Required reading:* Hellman, “Structuralism”; Shapiro, “Structure and Ontology”.

### **Week 9: Mathematical Explanation**

**Lecture 16.** (May 28th) Mathematical Explanation, 1/2

*Required reading:* Baker, “Are There Genuine Mathematical Explanations of Physical Phenomena?”.

**Lecture 17.** (May 30th) Mathematical Explanation, 2/2

*Required reading:* Leng, “Mathematical Explanation”.

**Quiz 6 due in class**

### **Week 10: Recap/spillover/TBA**

**Lecture 18.** (June 4th)

*Required reading:*

**Lecture 19.** (June 6th) Up for grabs! (You will be given choices.)

**Quiz 7 due in class**

*Required reading:* TBA

**Paper 2 due (online) on June 11th, 11:59pm**